Capactiors

Jessica Wade (jess.wade@kcl.ac.uk)

www.makingphysicsfun.com

Department of Physics & Centre for Plastic Electronics, Imperial College London

Faculty of Natural & Mathematical Sciences, King's College London

Electric Potential

- When there is a current in a wire:
  - Some electrons move freely
  - Apply a voltage = electric field = force acts on electrons
  - Force accelerates electrons = gain kinetic energy
  - Collide with atom (vibrational energy) = lose kinetic energy
  - Collisions with atoms = resistance in wire
  - Atoms vibrate more = wire heats up
  - Atoms vibrate more = resistance increases

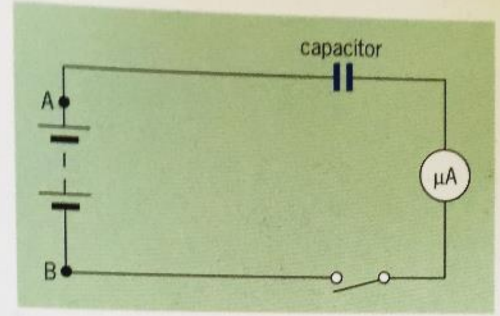


Fig 9.29 **A capacitor in a circuit**

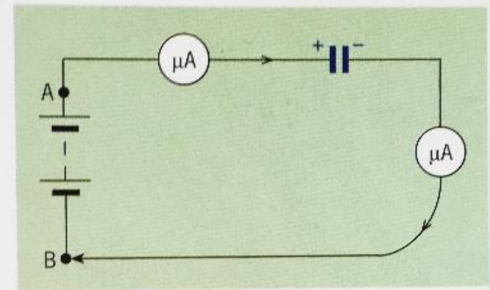


Fig 9.30(a) **Charging: A current is shown on both meters when the free wire is touched to side B of the battery to complete the circuit. The needle on both meters moves sharply to one side and quickly returns to zero. After this, no further current flows. The capacitor has been 'charged'**

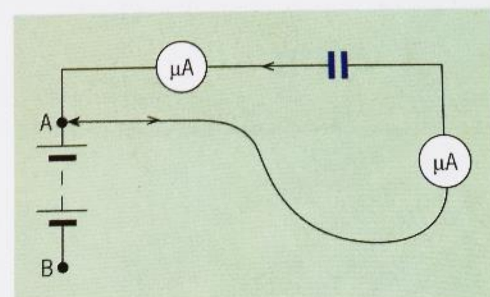


Fig 9.30(b) **Discharging: When the wire is moved from point B to point A, both meters show a current, but this time in the opposite direction. Again, the readings on the meters fall quickly to zero. The capacitor has now been 'discharged'**

Parallel Plate Capacitor

- Capacitance:

$$C = \frac{Q}{V} = \varepsilon_0 \frac{A}{d}$$

$$C = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

- ε_0 is the relative permittivity of the medium
- ε_R air = 1, paper = 2– 3, water = 80

Capacitors

- Example of a capacitor:
  - Flash on a camera bulb
- Battery drives current around a circuit
- One plate becomes positively charged, one becomes negatively charged
- Battery redistributes charge around the circuit
- Battery does work moving the charge and energy stored in a capacitor
- Discharge capacitor- stored energy does work moving the charges back

Parallel Plate Capacitor

- Electric field between two parallel plates can store charge (capacitor)
- Charge on plates ∝ potential difference CV
- Charge on plates ∝ area plates

$$\frac{Q}{A} \propto \frac{V}{d}$$

- Medium between plates (dielectric) is an insulator

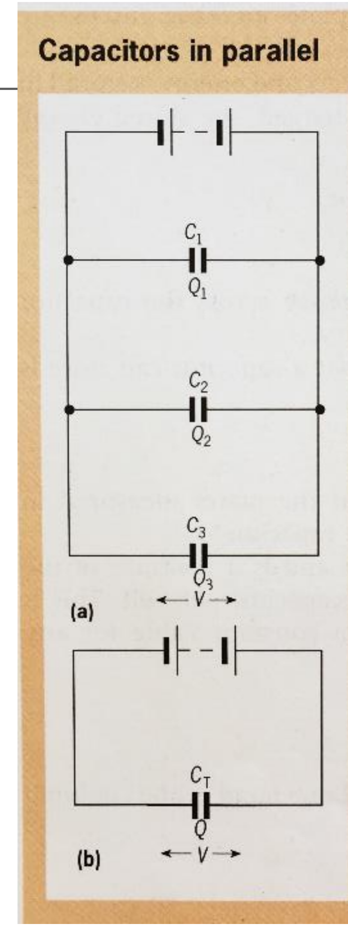
$$\frac{Q}{A} = \varepsilon_0 \frac{V}{d}$$

- ε_0 is the permittivity of free space, $[\varepsilon_0]$ = F m^{-1}
- A 1 farad capacitor charged by a potential difference of 1 volt carries a charge of 1 coloumb

Capacitors

- 1 Farad is a very large amount of capacitance
- Most capacitors are in micro, nano or picofards:
  - 1 μF = 10^{-6} F
  - 1 μF = 10^{3} nF
  - 1 μF = 10^{6} pF

Capacitors in Parallel

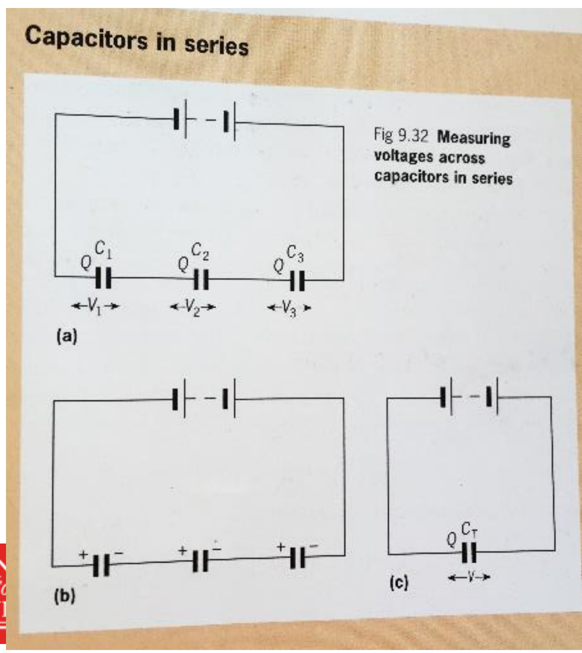


- As the number of capacitors connected in:
  - Parallel increases, capacitance increase

$$Q = Q_1 + Q_2 + Q_3$$
$$C_T V = C_1 V + C_2 V + C_3 V$$
$$C_T = C_1 + C_2 + C_3$$

  - Series increases, capacitance decreases

$$V = V_1 + V_2 + V_3$$
$$\frac{Q}{C_T} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$
$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Energy stored in a capacitor

- Get more charge into a capacitor requires you to *do work* against the repulsive forces between the charges
- Energy = joules/coulomb x coulomb = V × Q
- Charge on a capacitor ∝ Voltage
- Area = total work done charging capacitor

Area = 0.5 QV, Q = CV

Total Energy = $\frac{1}{2}QV = \frac{1}{2}CV^2$

- Hooke's Law: E = $\frac{1}{2}kx^2$

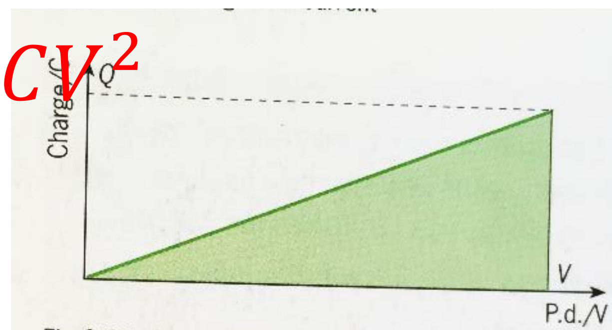


Fig 9.33 **Graph of charge on a capacitor against voltage**

King's College London · Imperial College London · WISE · CPE

Capacitors and Resistors

- Place a capacitor in series with a resistor:
  - Takes time for capacitor to charge and discharge because current in circuit decreases
  - Potential difference through resistor proportional to current > changes as capacitor charges and discharges
- Exponential decay: current in circuit decays by same ratio in successive equal intervals of time
  - Same as half-life of radioactive decay
  - Time constant = RC
  - $R \times C = \frac{Volts}{Amps} \times \frac{Coulombs}{Volts} = \frac{Coulombs}{\left(\frac{Coulombs}{Seconds}\right)} =$ seconds

The mathematics of exponential decay

$$\Delta Q = -I\Delta t$$

$$\Delta Q = -\frac{V}{R}\Delta t \quad \text{(where CV = Q)}$$

$$\Delta Q = -\frac{Q}{RC}\Delta t$$

$$\frac{\Delta Q}{\Delta t} = -\frac{Q}{RC}$$

$$\frac{dQ}{dt} = -\frac{Q}{RC}$$

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

The mathematics of exponential decay

$$\frac{dQ}{Q} = -\frac{dt}{RC}$$

$$\int_{Q_0}^{Q} \frac{dQ}{Q} = -\int_{0}^{t} \frac{dt}{RC}$$

$$\ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC}$$

$$\frac{Q}{Q_0} = exp\left(-\frac{t}{RC}\right)$$

$$Q = Q_0 exp\left(-\frac{t}{RC}\right)$$

Finding RC from a graph

- $ln\left(\frac{Q}{Q_0}\right) = -\frac{t}{RC} \rightarrow lnQ - lnQ_0 = -\frac{t}{RC}$
- Straight graph of *lnQ* against *t* is a straight line, with the intersect $-\frac{1}{RC}$