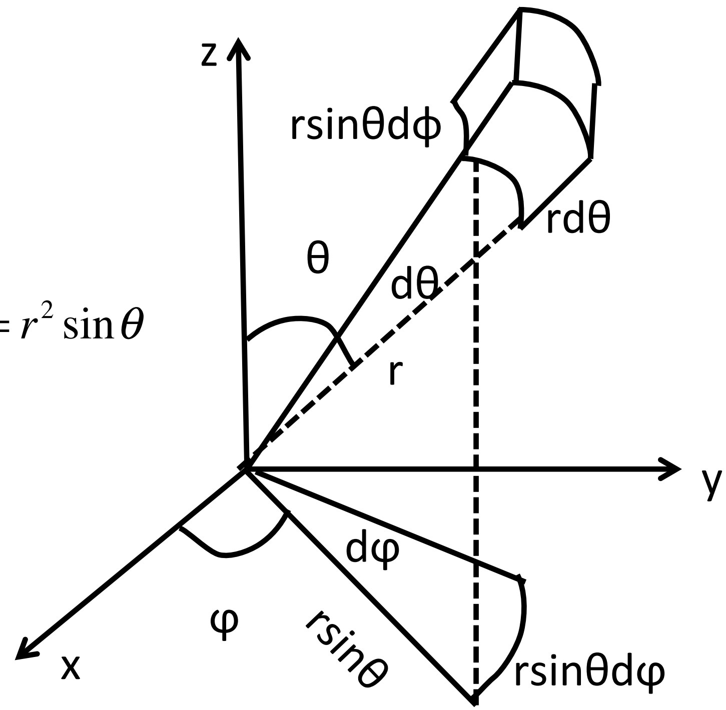
Spherical Polar Coordinates

x=rsinθ
y=rsinθsinφ
z=rcosθ

$$J = \left|\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}\right| = \begin{vmatrix} \sin\theta\cos\phi & \sin\phi\sin\theta & \cos\theta \\ r\cos\theta\cos\phi & \cos\theta\sin\phi & -r\sin\theta \\ -r\sin\theta\sin\phi & \sin\theta\cos\phi & 0 \end{vmatrix} = r^2\sin\theta$$

$$\iiint_R f\, dV = \iiint_R f r^2 \sin\theta\, d\theta\, d\phi\, dr$$



Surface Integrals

$$r = r_0 + \lambda A + \beta B$$

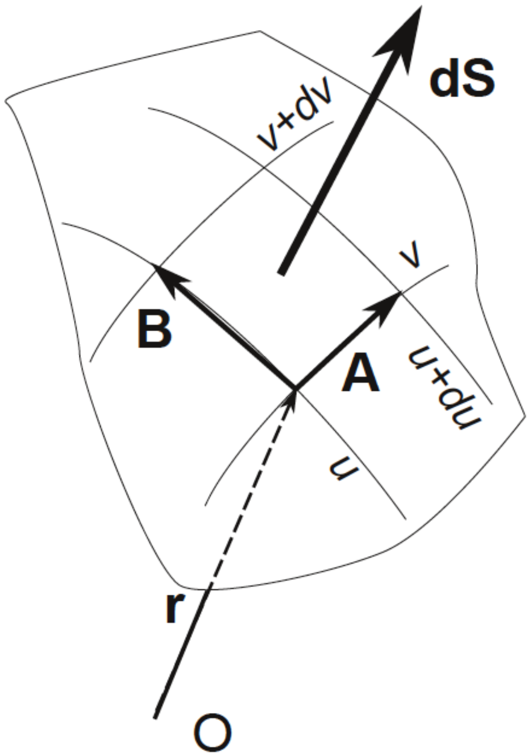
$$r = r(u, v)$$

$$\underline{dr} = \frac{\partial r}{\partial u} du + \frac{\partial r}{\partial v} dv$$

$$\underline{dS} = \left(\frac{\partial r}{\partial u} \times \frac{\partial r}{\partial v} \right) dudv = \underline{N} dudv = \hat{n} dS$$

$$\left| \underline{dS} \right| = Area$$

$$\hat{n} = \frac{\underline{N}}{\left| \underline{N} \right|}$$



Surface Integrals

Example: Surface of a sphere

$$r(\theta,\phi) = a\sin\theta\cos\phi\,\hat{i} + a\sin\phi\sin\theta\,\hat{j} + a\cos\theta\,\hat{k}$$

$$\underline{N} = a^2\sin\theta(\sin\theta\cos\phi\,\hat{i} + \sin\phi\sin\theta\,\hat{j} + \cos\theta\,\hat{k}) \equiv a^2\sin\theta\,\hat{r}$$

$$\iint |\underline{dS}| = \iint |\underline{N}| d\theta d\phi = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} a^2\sin\theta\, d\theta d\phi = 4\pi a^2$$

$$r = x(\theta,\phi)\hat{\underline{i}} + y(\theta,\phi)\hat{\underline{j}} + z(\theta,\phi)\hat{\underline{k}}$$

$$\frac{\partial r}{\partial\theta} = \ldots$$

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$$|\underline{dS}| = \left|\frac{\partial r}{\partial\theta}\times\frac{\partial r}{\partial\phi}\right| d\theta d\phi$$

$$S = \iint_R dS$$